unit-5

IV: Numerical Integration & The Solutions of Ordinary Differential Equation.

There are three rules
1. Tropizoidal Rule
2. Simpson $\frac{1}{3}$ rule
3. Simpson $\frac{3}{8}$ Rule

In Numerical integration, we solve the given problem by using the above rules.

Trapizoidal Rule:

$$\int_a^b y\,dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+y_3+\cdots+y_{n-1})\right]$$

Simpson $\frac{1}{3}$ Rule

$$\int_a^b y\,dx = \frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+y_5+\cdots)+2(y_2+y_4+y_6+\cdots)\right]$$

Simpson $\frac{3}{8}$ Rule

$$\int_a^b y\,dx = \frac{3h}{8}\left[(y_0+y_n)+3(y_1+y_2+y_4+y_5+y_7+y_8+\cdots)+2(y_3+y_6+y_9+\cdots)\right]$$

where $h=\frac{b-a}{n}$ and $y=f(x)$ is the given curve

1. Evaluate $\int_0^1 \frac{1}{1+x^2}dx$ where $n=4$

sol: $f(x)=\frac{1}{1+x^2}=y$; $a=0$; $b=1$, $n=4$; $h=\frac{b-a}{n}=\frac{1-0}{4}$

$h=\frac{1}{4}$

$x_0=a=0$

$\therefore y_0=f(x_0)=f(0)=\frac{1}{1+0^2}=1$

$x_1=x_0+h$
$=0+\frac{1}{4}$
$=\frac{1}{4}$

$y_1=f(x_1)$
$=\frac{1}{1+x_1^2}$; $x_1=\frac{1}{4}$
$=\frac{1}{1+(\frac{1}{4})^2}=\frac{16}{17}$

$x_2 = x_1 + h = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$y_2 = f(x_2) = \frac{1}{1+x_2^2}$, $x_2 = \frac{1}{2}$

$= \frac{1}{1+\left(\frac{1}{2}\right)^2} = \frac{1}{1+\frac{1}{4}} = \frac{4}{5}$

$x_3 = x_2 + h = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

$y_3 = f(x_3)$.

$= \frac{1}{1+x_3^2}$, $x_3 = 3/4$

$= \frac{1}{1+\left(\frac{3}{4}\right)^2} = \frac{1}{1+\frac{9}{16}} = \frac{16}{25}$

$x_4 = x_3 + h = \frac{3}{4} + \frac{1}{4} = 1$

$y_4 = f(x_4) = \frac{1}{1+x_4^2}$, $x_4 = 1$

$= \frac{1}{1+1^2} = \frac{1}{1+1} = \frac{1}{2}$

$\therefore y_0 = 1$; $y_1 = 0.9412$; $y_2 = 0.8$; $y_3 = 0.64$; $y_4 = 0.5$

① Trapizoidal Rule

$$\int_a^b y\,dx = \frac{h}{2}\left[(y_0+y_4) + 2(y_1+y_2+y_3)\right]$$

$$= \frac{\frac{1}{4}}{2}\left[(1+0.5) + 2(0.9412 + 0.8 + 0.64)\right]$$

$$= \frac{1}{8}\left[1.5 + 2(2.3812)\right]$$

$$= \frac{1}{8}\left[1.5 + 4.7624\right]$$

$$= 0.7828$$

② Sympson $\frac{1}{3}$ Rule

$$\int_a^b y\,dx = \frac{h}{3}\left[(y_0+y_4) + 4(y_1+y_3) + 2(y_2 + \cancel{y_4})\right]$$

$$= \frac{\frac{1}{4}}{3}\left[(1+0.5) + 4(0.9412 + 0.64) + 2(0.8)\right]$$

$$= \frac{1}{12}\left[1.5 + 4(1.5812) + 1.6\right]$$

$$= \frac{1}{12}\left[1.5 + 6.3248 + 1.6\right]$$

$$= \frac{1}{12}\left[9.4248\right]$$

$$= 0.7854$$

③ Sympson $\frac{3}{8}$ Rule

$$\int_a^b y\,dx = \frac{3h}{8}\left[(y_0+y_4)+3(y_1+y_2)+2(y_3)\right]$$

$$= \frac{3\cdot\frac{1}{4}}{8}\left[(1+0.5)+3(0.9412+0.8)+2(0.64)\right]$$

$$= \frac{3}{32}\left[1.5+3(1.7412)+1.28\right]$$

$$= \frac{3}{32}\left[1.5+5.2236+1.28\right]$$

$$= \frac{3}{32}\left[8.0036\right]$$

$$= \frac{24.0108}{32} = 0.7503375 = 0.7503$$

2. Evoluate $\int_1^2 \frac{1}{x}\,dx$ where $n=4$ $\quad h = \frac{b-a}{n} = \frac{2-1}{4} = \frac{1}{4}$

solu) $f(x) = \frac{1}{x}$, $n=4$, $a=1$, $b=2$

$x_0 = a = 1$; $y_0 = f(x_0) = \frac{1}{x_0} = \frac{1}{1} = 1$

$x_1 = x_0+h$ $\qquad y_1 = f(x_1)$

$= 1+\frac{1}{4}$ $\qquad = \frac{1}{x_1}$

$= \frac{5}{4}$ $\qquad = \frac{1}{\frac{5}{4}} = \frac{4}{5}$

$x_2 = x_1+h$ $\qquad y_2 = f(x_2)$

$= \frac{5}{4}+\frac{1}{4}$ $\qquad = \frac{1}{x_2}$

$= \frac{6}{4} = \frac{3}{2}$ $\qquad = \frac{1}{\frac{3}{2}} = \frac{2}{3}$

$x_3 = x_2+h$ $\qquad y_3 = f(x_3)$

$= \frac{3}{2}+\frac{1}{4}$ $\qquad = \frac{1}{x_3}$

$= \frac{6+1}{4}$ $\qquad = \frac{1}{\frac{7}{4}}$

$= \frac{7}{4}$ $\qquad = \frac{4}{7}$

$x_4 = x_3+h$ $\qquad y_4 = f(x_4)$

$= \frac{7}{4}+\frac{1}{4}$ $\qquad = \frac{1}{x_4}$

$= \frac{8}{4} = 2$ $\qquad = \frac{1}{2}$

$y_0 = 1,\ y_1 = 0.8\ ;\ y_2 = 0.6667;\ y_3 = 0.5714\ ;\ y_4 = 0.5$

① By Trapizoidal Rule

$$\int_a^b y\,dx = \frac{h}{2}\left[(y_0+y_4) + 2(y_1+y_2+y_3)\right]$$

$$= \frac{\frac{1}{4}}{2}\left[(1+0.5) + 2(0.8+0.6667+0.5714)\right]$$

$$= \frac{1}{8}\left[1.5 + 2(2.0384)\right]$$

$$= \frac{1}{8}\left[1.5 + 4.0768\right]$$

$$= \frac{5.5768}{8} = 0.4647333 = \cancel{0.4648} = 0.6971$$

2. Syjmpson $\frac{1}{3}$ Rule

$$\int_a^b y\,dx = \frac{h}{3}\left[(y_0+y_4) + 4(y_1+y_3) + 2y_2\right]$$

$$= \frac{\frac{1}{4}}{3}\left[(1+0.5) + 4(0.8+0.5714) + 2(0.6667)\right]$$

$$= \frac{1}{12}\left[1.5 + 4(1.3714) + 1.3334\right]$$

$$= \frac{1}{12}\left[1.5 + 5.4856 + 1.3334\right]$$

$$= \frac{8.319}{12} = 0.69325$$

3. Simpson $\frac{3}{8}$ Rule

$$\int_a^b y\,dx = \frac{3h}{8}\left[(y_0+y_4) + 3(y_1+y_2) + 2(y_3)\right]$$

$$= \frac{3\left(\frac{1}{4}\right)}{8}\left[(1+0.5) + 3(0.8+0.6667) + 2(0.5714)\right]$$

$$= \frac{3}{32}\left[1.5 + 3(1.4667) + 2(0.5714)\right]$$

$$= \frac{3}{32}\left[1.5 + 4.4001 + 1.1428\right]$$

$$= \frac{3}{32}\left[7.0429\right]$$

$$= \frac{21.1287}{32}$$

$$= 0.66027187\dot{5} = 0.6602$$

3. Evaluate $\int_0^1 \frac{1}{x+1}\,dx$, $n=5$

$n=5$ $a=0$, $b=1$ $h=\frac{b-a}{n}=\frac{1-0}{5}=\frac{1}{5}$

$y=f(x)=\frac{1}{x+1}$

$h=\frac{1}{5}$

$x_0=a=0$ $y_0=f(x_0)$ $y_1=f(x_1)$

$=\frac{1}{0+1}$ $=\frac{1}{\frac{1}{5}+1}$

$x_1=x_0+h$

$=0+\frac{1}{5}$ $=\frac{1}{1}$ $y_1=\frac{5}{6}$

$=\frac{1}{5}$ $=1$

$x_2=x_1+h$ $y_2=f(x_2)$ $x_5=x_4+h$

$=\frac{1}{5}+\frac{1}{5}$ $=\frac{1}{\frac{2}{5}+1}$ $=\frac{4}{5}+\frac{1}{5}$

$=1$

$=\frac{2}{5}$ $=\frac{5}{7}$ $y_5=f(x_4)$

$x_3=x_2+h$ $y_3=f(x_3)$ $=\frac{1}{1+1}$

$=\frac{2}{5}+\frac{1}{5}$ $=\frac{1}{\frac{3}{5}+1}$ $=\frac{1}{2}$

$=0.5$

$=\frac{3}{5}$ $=\frac{5}{8}$

$x_4=x_3+h$ $y_4=f(x_4)$

$=\frac{3}{5}+\frac{1}{5}$ $=\frac{1}{\frac{4}{5}+1}$

$=\frac{4}{5}$ $=\frac{5}{9}$

$x_0=0$, $y_0=1$; $y_1=0.8333$; $y_2=0.71438$; $y_5=0.5$; $h=0.2$

$y_3=0.625$; $y_4=0.5556$

① By Trapizoidal Rule.

$$\int_a^b y\,dx=\frac{h}{2}\left[(y_0+y_5)+2(y_1+y_2+y_3+y_4)\right]$$

$$=\frac{0.2}{2}\left[(1+0.5)+2(0.8333+0.71438+0.625+0.5556)\right]$$

$=0.1\,[1.5+2[2.7282]]$

$=0.1\,[1.5+5.4564]$

$=(0.1)(6.9564)$

$=0.69564$

② By simpson $\frac{1}{3}$ rule

$$\int_a^b y\,dx = \frac{h}{3}\left[(y_0+y_5)+4(y_1+y_3)+2(y_2+y_4)\right]$$

$$= \frac{0.2}{3}\left[(1+0.5)+4(0.8333+0.625)+2(0.7143+0.5556)\right]$$

$$= \frac{0.2}{3}\left[1.5+4(1.4583)+2(1.2699)\right]$$

$$= \frac{0.2}{3}\left[1.5+5.8332+2.5398\right]$$

$$= \frac{0.2}{3}\left[9.873\right]$$

$$= 0.2\,[3.291]$$

$$= 0.6582$$

③ By simpson $\frac{3}{8}$ Rule

$$\int_a^b y\,dx = \frac{3h}{8}\left[(y_0+y_5)+3(y_1+y_2+y_4)+2(y_3)\right]$$

$$= \frac{3(0.2)}{8}\left[(1+0.5)+3(0.8333+0.7143+0.5556)+2(0.625)\right]$$

$$= \frac{0.6}{8}\left[1.5+3(2.1032)+2(0.625)\right]$$

$$= \frac{0.6}{8}\left[1.5+6.3096+1.25\right]$$

$$= 0.6\,\frac{[9.0596]}{8}$$

$$= (0.6)(1.13245)$$

$$= 0.6795$$

4. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ $\quad n=6$

Solu) $y = f(x) = \frac{1}{1+x}$ $\quad a=0,\ b=6\ ;\ n=6 \quad h=\frac{b-a}{n}=\frac{6-0}{6}=1$

$x_0 = a = 0$, $\quad y_0 = f(x_0) = \frac{1}{1+0} = 1$

$x_1 = x_0+h = 1$ $\quad y_1 = f(x_1) = \frac{1}{1+1} = \frac{1}{2} = 0.5$

$x_2 = x_1+h = 1+1 = 2$ $\quad y_2 = f(x_2) = \frac{1}{1+2} = \frac{1}{3} = 0.3334$

$x_3 = x_2 + h$ $\quad y_3 = f(x_3)$ $\quad x_6 = x_5 + h$ $\quad y_6 = f(x_6)$

$= 2+1$ $\quad = \frac{1}{1+3}$ $\quad = 5+1$ $\quad = \frac{1}{1+6} = \frac{1}{7}$

$= 3$ $\quad = \frac{1}{4} = 0.25$ $\quad = 6$ $\quad = 0.1428$

$x_4 = x_3 + h$ $\quad y_4 = f(x_4)$

$= 3+1$ $\quad = \frac{1}{1+4} = \frac{1}{5} = 0.2$

$= 4$

$x_5 = x_4 + h$ $\quad y_5 = f(x_5)$

$= 4+1$ $\quad = \frac{1}{1+5} = \frac{1}{6} = 0.16667$

$= 5$

$x_0 = 0, \; y_0 = 1, \; y_1 = 0.5, \; y_2 = 0.3334, \; y_3 = 0.25; \; y_4 = 0.2$
$y_5 = 0.1667$
$y_6 = 0.1428$

① Trapizoidal Rule

$$\int_a^b y\,dx = \frac{h}{2}[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)]$$

$$= \frac{1}{2}[(1+0.1428) + 2(0.5+0.3334+0.25+0.2+0.1667)]$$

$$= 0.5[(1.1428) + 2(1.4501)]$$

$$= 0.5[1.1428 + 2.9002]$$

$$= 0.5[4.043]$$

$$= 2.0215$$

② simpson $\frac{1}{3}$ Rule.

$$\int_a^b y\,dx = \frac{3h}{8}[(y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4)]$$

$$= \frac{1}{3}[(1+0.1428) + 4(0.5+0.25+0.1667) + 2(0.3334+0.2)]$$

$$= 0.3334[1.1428 + 4(0.9167) + 2(0.5334)]$$

$$= 0.3334[1.1428 + 3.6668 + 1.0668]$$

$$= 0.3334[5.8764]$$

$$= 1.95919176$$

③ simpson $\frac{3}{8}$ Rule

$$\int_a^b y\,dx = \frac{3h}{8}\left[(y_0+y_6)+3(y_1+y_2+y_4+y_5)+2(y_3)\right]$$

$$= \frac{3(1)}{8}\left[(1+0.1428)+3(0.5+0.3334+0.2+0.1667)+2(0.25)\right]$$

$$= 0.375\left[1.1428+3(1.2001)+0.5\right]$$

$$= 0.375\left[1.1428+3.6003+0.5\right]$$

$$= 0.375\left[5.2431\right]$$

$$= 1.9661625$$

5. Evaluate $\int_0^4 e^x\,dx$ given $e = 2.72$ $e^2 = 7.39$ $e^3 = 20.09$

$e^4 = 54.6$; $n=4$

Solu) $y = f(x) = e^x$, $n=4$, $a=0$, $b=4$ $h = \frac{b-a}{n} = \frac{4-0}{4} = 1$

$x_0 = a = 0$ $y_0 = f(x_0) = e^0 = 1$

$x_1 = x_0 + h = 1$ $y_1 = f(x_1) = e^{x_1} = e^1 = 2.72$

$x_2 = x_1 + h = 1+1 = 2$ $y_2 = f(x_2) = e^{x_2} = e^2 = 7.39$

$x_3 = x_2 + h = 2+1 = 3$ $y_3 = f(x_3) = e^{x_3} = e^3 = 20.09$

$x_4 = x_3 + h = 3+1 = 4$ $y_4 = f(x_4) = e^{x_4} = e^4 = 54.6$

$x_0 = 0$; $y_0 = 1$; $y_1 = 2.72$; $y_2 = 7.39$, $y_3 = 20.09$; $y_4 = 54.6$

1) Trapizoidal Rule

$$\int_a^b y\,dx = \frac{h}{2}\left[(y_0+y_4)+2(y_1+y_2+y_3)\right]$$

$$= \frac{1}{2}\left[(1+54.6)+2(2.72+7.39+20.09)\right]$$

$$= 0.5\left[55.6+2(30.2)\right]$$

$$= 0.5\left[55.6+60.4\right]$$

$$= 0.5\left[116\right]$$

$$= 58$$

2)10(0.5
54.6
1.
20.09
7.

2) simpson $\frac{1}{3}$ rule:

$$\int_a^b y\,dx = \frac{h}{3}\left[(y_0+y_4)+4(y_1+y_3)+2(y_2)\right]$$

$$=\frac{1}{3}\left[(1+54.6)+4(2.72+20.09)+2(7.39)\right]$$

$$=\frac{1}{3}\left[55.6+4(22.81)+14.78\right]$$

$$=\frac{1}{3}\left[55.6+91.24+14.78\right]$$

$$=\frac{161.62}{3}$$

$$=53.8733$$

3) simpson $\frac{3}{8}$ Rule

$$\int_a^b y\,dx = \frac{3h}{8}\left[(y_0+y_4)+3(y_1+y_2)+2(y_3)\right]$$

$$=\frac{3(1)}{8}\left[(1+54.6)+3(2.72+7.39)+2(20.09)\right]$$

$$=\frac{3}{8}\left[55.6+3(10.11)+2(20.09)\right]$$

$$=0.375\left[55.6+30.33+40.18\right]$$

$$=0.375\left[126.11\right]$$

$$=47.29125$$

6. The velocity of a car running on a straight line at intravals of 2 minutes are given below.

Date 23/1/18

Time :	0	2	4	6	8	10	12
velocity :	0	22	30	27	18	7	0

find the distance covered by the car

Solu) Since we know that rate of change of displacement is called velocity. i.e., [Rate of change of velocity is called acceleration]

$$\frac{ds}{dt}=v$$

$$ds = v\,dt$$

$$s=\int_0^{12} v\,dt$$

1) Trapizoidal Rule

$s = \int_0^{12} v\,dt = \frac{h}{2}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$

$= \frac{2}{2}\left[(0+0) + 2(22+30+27+18+7)\right]$

$= 2[104]$

$= 208$

2) Simpson $\frac{1}{3}$ rd Rule

$s = \int_0^{12} v\,dt = \frac{h}{3}\left[(y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$

$= \frac{2}{3}\left[(0+0) + 4(22+27+7) + 2(30+18)\right]$

$= \frac{2}{3}\left[4(56) + 2(48)\right]$

$= \frac{2}{3}\left[224+96\right]$

$= \frac{2}{3}[320]$

$= 213.333$

3) Simpson $\frac{3}{8}$ Rule

$s = \int_0^{12} v\,dt = \frac{3h}{8}\left[(y_0+y_6) + 3(y_1+y_2+y_4+y_5) + 2(y_3)\right]$

$= \frac{3(\cancel{2})}{\cancel{8}_4}\left[(0+0) + 3(22+30+18+7) + 2(27)\right]$

$= \frac{3}{4}\left[3(77) + 54\right]$

$= \frac{3}{4}\left[231+54\right]$

$= \frac{3}{4} \times 285$

$= 213.75.$

7 The velocity v of a particle at a distance s from a point on its path. is given by the table below.

S	0	10	20	30	40	50	60
Velocity	47	58	64	65	61	52	38

Estimate the time taken to travel 60 m by using the Rules.

Solu] Since we know that the rate of change of displacement is called the velocity.

$$v = \frac{ds}{dt} \Rightarrow dt = \frac{1}{v}\,ds$$

$$t = \int_0^{60} \frac{1}{v}\,ds.$$

$\frac{1}{v} = \frac{1}{47} = 0.0212\,(y_0)$; $\frac{1}{58} = 0.0172\,(y_1)$; $\frac{1}{64} = 0.0156\,(y_2)$

$\frac{1}{65} = 0.0153\,(y_3)$; $\frac{1}{61} = 0.0164\,(y_4)$; $\frac{1}{52} = 0.0192\,(y_5)$

$\frac{1}{38} = 0.0263\,(y_6)$

1) Trapizoidal Rule

$$t = \int_0^{60} \frac{1}{v}\,ds = \frac{h}{2}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{10}{2}\left[(0.0212+0.0263) + 2(0.0172+0.0156+0.0154+0.0164+0.0192)\right]$$

$$= 5\left[(0.0475) + 2(\right.$$

$$= 5\left[(0.0475) + 0.1676\right]$$

$$= 1.0755 \text{ sec.}$$

2) Simpson $\frac{1}{3}$ Rule

$$t = \int_0^{60} \frac{1}{v}\,ds = \frac{h}{3}\left[(y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$$

$$= \frac{10}{3}\left[(0.0212+0.0263) + 4(0.0172+0.0154+0.0192) + 2(0.0156+0.0164)\right] \quad 0.0475$$

$$= \frac{10}{3}\left[(0.0475) + 0.2072 + 0.064\right]$$

$$= \frac{10}{3}\left[0.3187\right]$$

$$= 1.0623$$

$= \frac{h}{2} [253 \ldots]$

$= 1151.01$

Date 20/7/18

10 A river is 80 meters wide. The depth y of the river at a distance 'x' from one bank is given by the following table

x	0	10	20	30	40	50	60	70	80
y	0	4	7	9	12	15	14	8	3

find the approximate area of the cross section of the river. Find the cross section area of the given river.

solu) Since we know that the cross section area of the river is

$$A = \int_0^{80} y\,dx$$

1) By trapizoidal Rule.

$$\int_0^{80} y\,dx = \frac{h}{2}\left[(y_0 + y_8) + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7)\right]$$

$$= \frac{10}{2}\left[(0+80) + 2(10+20+30+40+50+60+70)\right]$$

$$= 5(80 + 2(280))$$

$$= 5[80+560]$$

$$= 5 \times 640$$

$$= 3200$$

$$= \frac{10}{2}\left[(0+3) + 2(4+7+9+12+15+14+8)\right]$$

$$= 5[3 + 2(69)]$$

$$= 5[3+138]$$

$$= 5[141] = 705 \text{ sq. units}$$

$141 \times 5 = 705$

2) Simpson $\frac{1}{3}$rd Rule

$$\int_0^{80} y\,dx = \frac{h}{3}\left[(y_0 + y_8) + 4(y_1 + y_3 + y_5 + y_7) + 2(y_2 + y_4 + y_6)\right]$$

$$= \frac{10}{3}\left[(0+3) + 4(4+9+15+14) + 2(7+12+14)\right]$$

$$= \frac{10}{3}\left[3 + 4(42) + 2(33)\right]$$

$$= \frac{10}{3}\left[3 + 168 + 66\right]$$

$$= \frac{10}{3} \times 237 = 710 \text{ sq. units}$$

3) Simpson $\frac{3}{8}$ Rule

$$\int_0^{8} y\,dx = \frac{3h}{8}\left[(y_0+y_8)+3(y_1+y_2+y_4+y_5+y_7)+2(y_3+y_6)\right]$$

$$= \frac{3(10)}{8}\left[(10+3)+3(4+7+12+15+8)+2(9+14)\right]$$

$$= \frac{30}{8}\left[3+3(46)+2(23)\right]$$

$$= \frac{\cancel{30}^{15}}{\cancel{8}_{4}}\left[3+138+46\right]$$

$$= \frac{15}{4}\times 167$$

$$= 701.25$$

11. A train is moving at the speed of 30 m/s. Suddenly breaks are applied. The speed of the train per second after t seconds is given by

time	0	5	10	15	20	25	30	35	40	45
speed	30 y_0	24 y_1	19 y_2	16 y_3	13 y_4	11 y_5	10 y_6	8 y_7	7 y_8	5 y_9

find the distance moved by the train in 45 seconds.

Solu) Since we know that

$$v = \frac{ds}{dt}$$

$$\Rightarrow ds = v\,dt$$

$$s = \int_0^{45} v\,dt \qquad h=5$$

1) Trapizoidal Rule

$$\int_0^{45} v\,dt = \frac{h}{2}\left[(y_0+y_9)+2(y_1+y_2+y_3+y_4+y_5+y_6+y_7+y_8)\right]$$

$$= \frac{5}{2}\left[(30+5)+2(24+19+16+13+11+10+8+7)\right]$$

$$= \frac{5}{2}\left[35+2(108)\right]$$

$$= \frac{5}{2}\left[35+216\right]$$

$$= \frac{5}{2}\left[251\right] = \frac{1255}{2}$$

$$= 627.5$$

2) Simpson $\frac{1}{3}$ Rule

$$\int_0^{45} v\,dt = \frac{h}{3}\left[(y_0+y_9)+4(y_1+y_3+y_5+y_7)+2(y_2+y_4+y_6+y_8)\right]$$

$$= \frac{5}{3}\left[(30+5)+4(\cancel{30}+24+16+11+8)+2(\cancel{198}+13+10+7)\right]$$

$$= \frac{5}{3}\left[(35)+4(\cancel{103}\ 59)+2(49)\right]$$

$$= \frac{5}{3}\left[35+\cancel{412}\ 236+98\right]$$

$$= \frac{5}{3}[\cancel{581}] = \frac{5}{\cancel{3}} \quad \frac{1845}{3}$$

$$= \cancel{885} \quad 615.$$

412
84
35
531

3) Simpson $\frac{3}{8}$ Rule

$$\int_0^{45} v\,dt = \frac{3h}{8}\left[(y_0+y_9)+3(y_1+y_2+y_4+y_5+y_7+y_8)+2(y_3+y_6)\right]$$

$$= \frac{3\times5}{8}\left[(30+5)+3(24+\cancel{198}+13+11+8+7)+2(16+10)\right]$$

$$= \frac{15}{8}\left[35+3(\cancel{161}\ 82)+2(26)\right] = \frac{15}{8}\left[35+483+52\right]$$

$$= \frac{15}{8}[\cancel{870}\ 333] = \cancel{1068.75} = \frac{4995}{8} = 624.375.$$

12. In an experiment, a quantity, G was measured as follows. Solu) G(26) = 101.24

G(20) = 95.9 compute $\int_{20}^{26} G(x)\,dx$

G(21) = 96.85

G(22) = 97.77

G(23) = 98.68

G(24) = 99.56

G(25) = 100.41

G(26) =

Solu)

x	20	21	22	23	24	25	26
y	95.9	96.85	97.77	98.68	99.56	100.41	101.24

1) Given G(26) = 101.24

1) Trapizoidal Rule

$$\int_{20}^{26} G(x)\,dx = \frac{h}{2}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{1}{2}\left[(95.9+101.24) + 2(96.85+97.77+98.68+99.56+100.41)\right]$$

$$= 0.5\left[197.14 + 2(493.27)\right]$$

$$= 0.5\left[197.14 + 986.54\right]$$

$$= 0.5\left[1183.68\right]$$

$$= 591.84$$

2) Simpson $\frac{1}{3}^{rd}$ Rule

$$\int_{20}^{26} G(x)\,dx = \frac{h}{3}\left[(y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$$

$$= \frac{1}{3}\left[(95.9+101.24) + 4(96.85+98.68+100.41) + 2(97.77+99.56)\right]$$

$$= 0.3333\left[197.14 + 4(295.94) + 2(197.33)\right]$$

$$= 0.3333\left[197.14 + 1183.76 + 394.66\right]$$

$$= 0.3333\left[1775.56\right]$$

$$= 591.794148$$

3) Simpson $\frac{3}{8}$ Rule

$$\int_{20}^{26} G(x)\,dx = \frac{3h}{8}\left[(y_0+y_6) + 3(y_1+y_2+y_4+y_5) + 2(y_3)\right]$$

$$= \frac{3}{8}\left[(95.9+101.24) + 3(96.85+97.77+99.56+100.41) + 2(98.68)\right]$$

$$= 0.375\left[197.14 + 3(394.59) + 2(98.68)\right]$$

$$= 0.375\left[197.14 + \cancel{1882.54}\ 1183.77 + 197.36\right]$$

$$= 0.375\left[1578.27\right]$$

$$= 591.85125$$

13 The speed of a train at varies times after leaving one station until it stops at another station are given in the following table

speed :	0^{y_0}	13^{y_1}	33^{y_2}	39.5^{y_3}	40^{y_4}	40^{y_5}	36^{y_6}	15^{y_7}	0^{y_8}
time :	0	0.5	1	1.5	2	2.5	3	3.5	4

find the distance between the two stations.

solu) since we know that the rate of change of displacement is called velocity

$$\frac{ds}{dt} = v$$

$$ds = v\,dt$$

$$s = \int_0^4 v\,dt$$

1) Trapizoidal Rule

$$\int_0^4 v\,dt = \frac{h}{2}\left[(y_0+y_8) + 2(y_1+y_2+y_3+y_4+y_5+y_6+y_7)\right]$$

$$= \frac{0.5}{2}\left[(0+0) + 2(13+33+39.5+40+40+36+15)\right]$$

$$= \frac{0.5}{2}\left[2(216.5)\right]$$

$$= 0.25\left[433\right]$$

$$= 108.25$$

2) Simpson $\frac{1}{3}$rd Rule

$$\int_0^4 v\,dt = \frac{h}{3}\left[(y_0+y_8) + 4(y_1+y_3+y_5+y_7) + 2(y_2+y_4+y_6)\right]$$

$$= \frac{0.5}{3}\left[(0+0) + 4(13+39.5+40+15) + 2(33+40+36)\right]$$

$$= \frac{\cancel{0.16667}\ 0.5}{3}\left[4(107.5) + 2(109)\right]$$

$$= \frac{0.5}{3}\ \cancel{0.05556}\left[430 + 218\right]$$

$$= \cancel{0.05556}\ \frac{0.5}{3}\left[648\right]$$

$$= \cancel{36}.\ \frac{324}{3} = 108$$

3) Simpson $\frac{3}{8}$ Rule

$$\int_0^4 v\,dt = \frac{3h}{8}\left[(y_0+y_8) + 3(y_1+y_2+y_4+y_5+y_7) + 2(y_3+y_6)\right]$$

$$= \frac{3(0.5)}{8}\left[(0+0) + 3(13+33+40+40+15) + 2(39.5+36)\right]$$

$= 0.1875[(141)3 + 2(75.5)]$

$= 0.1875[423 + 151]$

$= 0.1875(574)$

$= 107.625$

14. Evaluate $\int_0^6 \frac{dx}{1+x^4}$ $n = 6$

Solu) Given

$\int_0^6 \frac{dx}{1+x^4}$; $n=6$; $h = \frac{b-a}{n} = \frac{6-0}{6} = 1$

$x_0 = a = 0$ $\quad y_0 = f(x_0) = \frac{1}{1+(0)^4} = \frac{1}{1} = 1 = 1$

$x_1 = x_0 + h = 0+1 = 1$ $\quad y_2 = f(x_2) = \frac{1}{1+2^4} = \frac{1}{1+16} = \frac{1}{17} = 0.05882$

$x_2 = x_1 + h = 1+1 = 2$ $\quad y_3 = f(x_3) = \frac{1}{1+3^4} = \frac{1}{1+81} = \frac{1}{82} = 0.012195$

$\quad y_4 = f(x_4) = \frac{1}{1+4^4} = \frac{1}{1+256} = \frac{1}{257} = 0.003891$

$x_3 = x_2 + h = 2+1 = 3$ $\quad y_5 = f(x_5) = \frac{1}{1+5^4} = \frac{1}{1+625} = \frac{1}{626} = 0.001597$

$\quad y_1 = f(x_1) = \frac{1}{1+1} = \frac{1}{2} = 0.5$

$x_4 = x_3 + h = 3+1 = 4$ $\quad x_5 = x_4 + h = 4+1 = 5$ $\quad y_6 = f(x_6) = \frac{1}{1+6^4} = \frac{1}{1+1296} = \frac{1}{1297} = 0.00077101$

$x_6 = x_5 + h = 5+1 = 6$

1) Trapizoidal Rule

$$\int_a^b y\,dx = \frac{h}{2}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{1}{2}\left[(1+0.00071) + 2(0.5+0.05882+0.012195+0.003891+0.001597)\right]$$

$$= \frac{1}{2}\left[1.00071 + 2(0.576503)\right]$$

$$= 0.5[1.00071 + 1.153006]$$

$$= 0.5[2.153716]$$

$$= 1.076858$$

2) Simpson $\frac{1}{3}$rd Rule.

$$\int_a^b y\,dx = \frac{h}{3}\left[(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)\right]$$

$$= \frac{1}{3}\left[(1+0.00071)+4(0.5+0.012195+0.001597)\right.$$
$$\left.+2(0.05882+0.003891)\right]$$

$$= \frac{1}{3}\left[1.00071+4(0.513792)+2(0.062711)\right]$$

$$= \frac{1}{3}\left[1.00071+2.055168+0.125422\right]$$

$$= \frac{1}{3}\left[3.1813\right]$$

$$= 1.06043333$$

3) Simpson $\frac{3}{8}$ Rule

$$\int_a^b y\,dx = \frac{3h}{8}\left[(y_0+y_6)+3(y_1+y_2+y_4+y_5)+2(y_3)\right]$$

$$= \frac{3}{8}\left[(1+0.00071)+3(0.5+0.05882+0.003891+0.001597)\right.$$
$$\left.+2(0.012195)\right]$$

$$= 0.375\left[1.00071+3(0.564308)+2(0.012195)\right]$$

$$= 0.375\left[1.00071+1.692924+0.02439\right]$$

$$= 0.375\left[2.718024\right]$$

$$= 1.019259$$

Date 26/7/18

5. Find the value of $\log_e 2$ from $\int_0^1 \frac{x^2}{1+x^3}dx$ by using Simpson $\frac{1}{3}$rd Rule by dividing the range of integration into 4 equal parts

Sol) Given $\int_0^1 \frac{x^2}{1+x^3}dx$

$$a=0,\ b=1,\ n=4,\ h=\frac{b-a}{n}=\frac{1}{4}=0.25$$

$$y=\frac{x^2}{1+x^3}$$

$$x_0=0 \Rightarrow y_0=\frac{x_0^2}{1+x_0^3}=\frac{0}{1+0}=0$$

$$x_1=x_0+h \qquad y_1=\frac{x_1^2}{1+x_1^3}=\frac{(0.25)^2}{1+(0.25)^3}=\frac{0.0625}{1+0.015625}=\frac{0.0625}{1.015625}$$
$$=0+0.25$$
$$=0.25 \qquad\qquad =0.061538$$

$x_2 = x_1 + h$ $\quad y_2 = \frac{x_2^2}{1+x_2^3} = \frac{(0.5)^2}{1+(0.5)^3} = \frac{0.25}{1+0.125} = \frac{0.25}{1.125}$

$= 0.25 + 0.25$

$= 0.5$ $\quad = 0.2222$

$x_3 = x_2 + h$ $\quad y_3 = \frac{x_3^2}{1+x_3^3} = \frac{(0.75)^2}{1+(0.75)^3} = \frac{0.5625}{1+0.421875} = \frac{0.5625}{1.421875}$

$= 0.5 + 0.25$

$= 0.75$ $\quad = 0.3956$

$x_4 = x_3 + h$ $\quad y_4 = \frac{x_4^2}{1+x_4^3} = \frac{1^2}{1+1^3} = \frac{1}{2} = 0.5$

$= 0.75 + 0.25$

$= 1$

$$\begin{array}{r} 0.0615 \\ 0.3956 \\ \hline 0.4571 \end{array}$$

Simpson $\frac{1}{3}$rd Rule

$$\int_0^1 y\,dy = \frac{h}{3}\left[(y_0+y_4) + 4(y_1+y_3) + 2y_2\right]$$

$$= \frac{0.25}{3}\left[(0+0.5) + 4[(0.0615) + 0.3956] + 2(0.2222)\right]$$

$$= \frac{0.25}{3}\left[0.5 + 4(0.4571) + 2(0.2222)\right]$$

$$= \frac{0.25}{3}\left[0.5 + 1.8284 + 0.4444\right]$$

$$\begin{array}{r} 0.4444 \\ 1.8284 \\ 0.5 \\ \hline 2.7724 \end{array}$$

$$= \frac{0.25}{3}\left[2.7724\right]$$

$$= 0.231066$$

$$= 0.2311$$

16. Find an approximate value of $\log_e 5$ by calculating $\int_0^5 \frac{1}{4x+5}dx$ by simpson $\frac{1}{3}$rd Rule to four decimal places by dividing the range into 10 equal parts

Solu) Given $\int_0^5 \frac{1}{4x+5}$ $\quad a = 0,\ b = 5,\ n = 10,\ h = \frac{b-a}{n} = \frac{5-0}{10} = \frac{1}{2} = 0.5$

$y = x_0 = 0$ $\quad y_0 = \frac{1}{4x_0+5}$

$x_1 = x_0 + h$

$= 0 + 0.5$ $\quad y_0 = \frac{1}{4(0)+5}$

$= 0.5$

$y_0 = \frac{1}{5} = 0.2$

$x_2 = x_1 + h$ $\quad y_2 = \frac{1}{4x_1+5}$

$= 0.5 + 0.5$

$= 1.0$ $\quad y_1 = \frac{1}{4(0.5)+5} = \frac{1}{2+5} = \frac{1}{7} = 0.1428$

$x_3 = x_2 + h$
$= 1.0 + 0.5$
$= 1.5$

$y_2 = \frac{1}{4x_2+5} = \frac{1}{4(1)+5} = \frac{1}{4+5} = \frac{1}{9} = 0.1111$

$x_4 = x_3 + h$
$= 1.5 + 0.5$
$= 2$

$y_3 = \frac{1}{4x_3+5} = \frac{1}{4(1.5)+5} = \frac{1}{6+5} = \frac{1}{11} = 0.0909$

$y_4 = \frac{1}{4x_4+5} = \frac{1}{4(2)+5} = \frac{1}{13} = 0.0769$

$x_5 = x_4 + h$
$= 2 + 0.5$
$= 2.5$

$y_5 = \frac{1}{4x_5+5} = \frac{1}{4(2.5)+5} = \frac{1}{10+5} = \frac{1}{15} = 0.0667$

$x_6 = x_5 + h$
$= 2.5 + 0.5$
$= 3.0$

$y_6 = \frac{1}{4x_6+5} = \frac{1}{4(3.0)+5} = \frac{1}{17} = 0.0588$

$x_7 = x_6 + h$
$= 3 + 0.5$
$= 3.5$

$y_7 = \frac{1}{4x_7+5} = \frac{1}{4(3.5)+5} = \frac{1}{14+5} = \frac{1}{19} = 0.05263$

$x_8 = x_7 + h$
$= 3.5 + 0.5$
$= 4.0$

$y_8 = \frac{1}{4x_8+5} = \frac{1}{4(4)+5} = \frac{1}{21} = 0.0476$

$x_9 = x_8 + h$
$= 4.0 + 0.5$
$= 4.5$

$y_9 = \frac{1}{4x_9+5} = \frac{1}{4(4.5)+5} = \frac{1}{18+5} = \frac{1}{23} = 0.0435$

$x_{10} = x_9 + h$
$= 4.5 + 0.5$
$= 5$

$y_{10} = \frac{1}{4x_{10}+5} = \frac{1}{4(5)+5} = \frac{1}{20+5} = \frac{1}{25} = 0.04$

Simpson $\frac{1}{3}$rd Rule

$$\int_0^5 y\,dx = \frac{h}{3}\left[(y_0+y_{10}) + 4(y_1+y_3+y_5+y_7+y_9) + 2(y_2+y_4+y_6+y_8)\right]$$

$$= \frac{0.5}{3}\left[(0.2+0.04) + 4(0.1429+0.0909+0.0667+0.05263+0.0435) + 2(0.1111+0.0769+0.0588+0.0476)\right]$$

$$= \frac{0.5}{3}\left[0.24 + 4(0.39663) + 2(0.2943)\right]$$

$$= \frac{0.5}{3}\left[0.24 + 1.58652 + 05.892\right]$$

$$= \frac{0.5}{3}\left[2.41572\right] = 0.40253 = 0.4025$$

17. Evaluate $\int_0^2 \frac{dx}{x^2+x+1}$ to three decimal dividing the range into eight equal parts.

solu) Given $a=0$, $b=2$, $n=8$; $h = \frac{b-a}{n} = \frac{2-0}{8} = \frac{2}{8} = 0.25$

$y = f(x) = \frac{1}{x^2+x+1}$

$x_0 = a = 0 \Rightarrow y_0 = \frac{1}{x_0^2+x_0+1} = \frac{1}{0^2+0+1} = \frac{1}{1} = 1$

$x_1 = x_0+h$ $\quad y_1 = \frac{1}{x_1^2+x_1+1}$

$= 0+0.25$ $\quad y_1 = \frac{1}{(0.25)^2+(0.25)+1} = 0.7619$

$= 0.25$

$x_2 = x_1+h$ $\quad y_2 = \frac{1}{x_2^2+x_2+1} = \frac{1}{(0.5)^2+0.5+1} = \frac{1}{1.75} = 0.5714$

$= 0.25+0.25$

$= 0.5$

$x_3 = x_2+h$ $\quad y_3 = \frac{1}{x_3^2+x_3+1}$

$= 0.5+0.25$ $\quad = \frac{1}{(0.75)^2+0.75+1} = 0.4324$

$= 0.75$

$x_4 = x_3+h$ $\quad y_4 = \frac{1}{x_4^2+x_4+1} = \frac{1}{1+1+1} = \frac{1}{3} = 0.3333$

$= 0.75+0.25$

$= 1$

$x_5 = x_4+h$ $\quad y_5 = \frac{1}{x_5^2+x_5+1} = \frac{1}{(1.25)^2+1.25+1} = 0.2602$

$= 1+0.25$

$= 1.25$ $\quad y_6 = \frac{1}{x_6^2+x_6+1} = \frac{1}{(1.5)^2+1.5+1} = 0.2105$

$x_6 = x_5+h$

$= 1.25+0.25$

$= 1.5$ $\quad y_7 = \frac{1}{x_7^2+x_7+1} = \frac{1}{(1.75)^2+1.75+1} = 0.17204$

$x_7 = x_6+h$

$= 1.5+0.25$

$= 1.75$ $\quad y_8 = \frac{1}{x_8^2+x_8+1} = \frac{1}{2^2+2+1} = \frac{1}{7} = 0.14285$

$x_8 = x_7+h$

$= 1.75+0.25$

$= 2$

Simpson $\frac{1}{3}$rd Rule

$$\int_0^2 y\,dx = \frac{h}{3}\left[(y_0+y_8)+4(y_1+y_3+y_5+y_7)+2(y_2+y_4+y_6)\right]$$

$$= \frac{0.25}{3}\left[(1+0.14288)+4(0.7619+0.4324+0.2622+0.17204)\right.$$
$$\left.+2(0.5714+0.3333+0.2105)\right]$$

$$= \frac{0.25}{3}\left[1.1428+2(1.1149)+4(1.62854)\right]$$

$$= \frac{0.25}{3}\left[1.1428+2.2298+6.51416\right]$$

$$= \frac{0.25}{3}\left[9.88676\right] = \frac{2.47184}{3}$$

$$= 0.824$$

18. Evaluate $\int_0^6 \frac{x}{1+x^5}$ by using simpson $\frac{3}{8}$ Rule where $n=6$

H.W

Solu) Given that $a=0,\ b=6,\ n=6$; $h=\frac{b-a}{n}=\frac{6-0}{6}=1$

$$y=f(x)=\frac{x}{1+x^5}$$

$x_0=a=0,\quad y_0=\frac{x_0}{1+x_0^5}=\frac{0}{1+0}=0$

$x_1=x_0+h \quad y_1=\frac{x_1}{1+x_1^5}=\frac{1}{1+1}=\frac{1}{2}=0.5$

$=0+1$

$=1$

$x_2=x_1+h \quad y_2=\frac{2}{1+2^5}=\frac{2}{1+32}=\frac{2}{33}=\cancel{0.0303}\ 0.0606$

$=1+1$

$=2$

$x_3=x_2+h \quad y_3=\frac{3}{1+3^5}=\frac{3}{1+243}=\frac{3}{244}=\cancel{0.0040983}\ 0.0122$

$=2+1$

$=3$

$y_4=\frac{x_4}{1+x_4^5}=\frac{4}{1+4^5}=\frac{4}{1+1024}=\frac{1}{1025}=\cancel{0.000975}\ 0.003902$

$x_4=x_3+h$

$=3+1$

$=4$

$y_5=\frac{x_5}{1+x_5^5}=\frac{5}{1+5^5}=\frac{5}{1+3125}=\frac{5}{3126}=\cancel{0.0003198}\ 0.001599$

$x_5=x_4+h$

$=4+1$

$=5$

$y_6=\frac{x_6}{1+x_6^6}=\frac{6}{1+6^6}=\frac{6}{1+7776}=\frac{6}{7777}=\cancel{0.00012858}\ 0.007722$

$x_6=x_5+h$

$=5+1$

$=6$

$2\times2=4$
$2\times2=4$
$2=2$
32

Simpson $\frac{3}{8}$ Rule

$$\int_0^6 y\,dx = \frac{3h}{8}\left[(y_0+y_6)+4(y_1+y_2+y_4+y_5)+2(y_3)\right]$$

$$= \frac{3(1)}{8}\left[(0+0.000128)+4(0.5+0.0040+0.000319)+2(0.0303+0.000975)\right]$$

wrong

$$= \frac{3}{8}\left[0.000128+4(0.504319)+2(0.031275)\right]$$

$$= 0.375[0.000128+2.017276+0.06255]$$

$$= 0.375[2.079954]$$

$$= 0.77998$$

$$= \frac{3}{8}[0+0.0077]+3[0.5+0.0606+0.0039+0.0015]$$

$$+2[0.0122]$$

$$= \frac{3}{8}[0.0077+3(0.566)+0.0244]$$

$$= \frac{3}{8}[0.0077+1.698+0.0244]$$

$$= \frac{3}{8}[1.7301]$$

$$= \frac{5.1903}{8} = 0.6487875$$

19. Evaluate $\int_0^1 \frac{dx}{1+x}$ by using Simpson $\frac{1}{3}$rd Rule where $h=0.2$

x	0 (x_0)	0.2 (x_1)	0.4 (x_2)	0.6 (x_3)	0.8 (x_4)	1 (x_5)

and also find $\log_e 2$

Solu) Given that

$$\int_0^1 \frac{dx}{1+x}$$

put $1+x = t$

$dx = dt$

$x=0,\ t=1+x=1+0=1$

$x=1,\ t=1+x=1+1=2$

$$\int_0^1 \frac{dx}{1+x} = \int_1^2 \frac{dt}{t}$$

$$= [\log t]_1^2$$
$$= \log_e^2 - \log_e^1$$
$$= \log_e^2 - 0$$
$$\int_0^2 \frac{dx}{1+x} = \log_e^2 \rightarrow ①$$

By simpson $\frac{1}{3}$rd Rule

$$\int_0^1 \frac{1}{1+x}\,dx = \frac{h}{3}\left[(y_0+y_5) + 4(y_1+y_3) + 2(y_2+y_4)\right]$$

$h = 0.2$

$$y = f(x) = \frac{1}{1+x}$$

$x_0 = a = 0$

$x_1 = x_0+h = 0+0.2 = 0.2$

$x_2 = x_1+h = 0.2+0.2 = 0.4$

$x_3 = x_2+h = 0.4+0.2 = 0.6$

$x_4 = x_3+h = 0.6+0.2 = 0.8$

$x_5 = x_4+h = 0.8+0.2 = 1.0$

$$y_0 = \frac{1}{1+x_0} = \frac{1}{1+0} = \frac{1}{1} = 1$$
$$y_1 = \frac{1}{1+x_1} = \frac{1}{1+0.2} = \frac{1}{1.2} = 0.8333$$
$$y_2 = \frac{1}{1+x_2} = \frac{1}{1+0.4} = \frac{1}{1.4} = 0.71428$$
$$y_3 = \frac{1}{1+x_3} = \frac{1}{1+0.6} = \frac{1}{1.6} = 0.625$$
$$y_4 = \frac{1}{1+x_4} = \frac{1}{1+0.8} = \frac{1}{1.8} = 0.5555$$
$$y_5 = \frac{1}{1+x_5} = \frac{1}{1+1} = \frac{1}{2} = 0.5$$

$$= \frac{0.2}{3}\left[(1+0.5) + 4(0.8333+0.625) + 2(0.71428+0.5555)\right]$$
$$= 0.0666\left[1.5 + 4(1.4583) + 2(1.26978)\right]$$
$$= 0.0666\left[1.5 + 5.8332 + 2.53958\right]$$
$$= 0.0666\left[9.87276\right]$$
$$= 0.658118$$

20. Evaluate $\int_0^5 \frac{dx}{4x+5}$ by using simpson $\frac{1}{3}$rd Rule where $h=1$ and also find $\log_e^5$

Solu) $\int_0^5 \frac{dx}{4x+5}$.

put $4x+5 = t$

$4\,dx = dt$

$dx = \frac{dt}{4}$

Put $\cancel{t}$. $x=0$ $t = 4(0)+5 = 5$

$x=5$ $t = 4(5)+5 = 25$

$$\int_0^5 \frac{dx}{4x+5} = \int_5^{25} \frac{dt}{4t} = \frac{1}{4}\int_5^{25} \frac{dt}{t}$$

$$= \frac{1}{4}\left[\log t\right]_5^{25}$$

$$= \frac{1}{4}\left[\log_e^{25} - \log_e^5\right]$$

$$= \frac{1}{4}\left[2\log_e^5 - \log_e^5\right)$$

$$= \frac{1}{4}\left[2-1\right]\log_e^5$$

$$\therefore \int_0^5 \frac{dx}{4x+5} = \frac{1}{4}\log_e^5 \rightarrow (1) \text{ (or) } \log_e^5 = 4\int_0^5 \frac{dx}{4x+5}$$

NOW

$F(x) = \frac{1}{4x+5}$ $\quad a = 0 = x_0$

$x_1 = x_0 + h$ $\quad x_2 = x_1 + h$ $\quad x_3 = x_2 + h$ $\quad x_4 = x_3 + h$ $\quad x_5 = x_4 + h$

$= 0+1$ $\quad = 1+1$ $\quad = 2+1$ $\quad = 3+1$ $\quad = 4+1$

$= 1$ $\quad = 2$ $\quad = 3$ $\quad = 4$ $\quad = 5$

$y_0 = \frac{1}{4x_0+5} = \frac{1}{4(0)+5} = \frac{1}{5} = 0.2$

$y_1 = \frac{1}{4x_1+5} = \frac{1}{4(1)+5} = \frac{1}{9} = 0.1111$

$y_2 = \frac{1}{4x_2+5} = \frac{1}{4(2)+5} = \frac{1}{13} = 0.07692$

$y_3 = \frac{1}{4x_3+5} = \frac{1}{4(3)+5} = \frac{1}{17} = 0.05882$

$y_4 = \frac{1}{4x_4+5} = \frac{1}{4(4)+5} = \frac{1}{21} = 0.04761$

$y_5 = \frac{1}{4x_5+5} = \frac{1}{4(5)+5} = \frac{1}{25} = 0.04$

Simpson $\frac{1}{3}$rd Rule

$$\int_0^5 \frac{dx}{4x+5} = \frac{h}{3}\left[(y_0+y_5)+4(y_1+y_3)+2(y_2+y_4)\right]$$

$$= \frac{1}{3}\left[(0.2+0.04)+4(0.1111+0.0588)+2(0.0769+0.04761)\right]$$

$$= \frac{1}{3}\left[(0.2+0.04)+4(0.1699)+2(0.12451)\right]$$

$$= \frac{1}{3}\left[0.24+4(0.1699)+2(0.12451)\right]$$

$$= \frac{1}{3}\left[0.24+0.6796+0.24902\right]$$

$$= 0.3333\left[1.16862\right]$$

$$= 0.389501 \rightarrow (2)$$

$$\frac{1}{4}\log_e 5 = 0.389501 \quad [\text{from } (1) \,\&\, (2)]$$

$$\log_e 5 = 4 \times 0.389501$$

$$= 1.558004$$

Date 30/7/18

Numerical Solution for the Ordinary differential Equations.

Picards Method:

Consider $\frac{dy}{dx} = F(x,y)$ then

$y_1^{(1)} = y_0 + \int_{x_0}^{x} F(x, y_0)\,dx$ is called first approximation

$y^{(2)} = y_0 + \int_{x_0}^{x} F(x, y^{(1)})\,dx$ is called second approximation

$y^{(3)} = y_0 + \int_{x_0}^{x} F(x, y^{(2)})\,dx$ is called third approximation

$y^{(4)} = y_0 + \int_{x_0}^{x} F(x, y^{(3)})\,dx$ is called fourth approximation

1) Using picards method to find the value of y and $x=0.1$, $x=0.2$ given $\frac{dy}{dx}=x-y$ if intial condation $y=1$ when $x=0$

Solu) Given $\frac{dy}{dx} = x-y$ if initial condition $y=1$ when $x=0$

solu) $f(x,y) = x-y \rightarrow (1)$

Given

$y=1$, when $x=0 \Rightarrow x_0=0,\ y_0=1$

By picards method

$$y^{(1)} = y_0+\int_{x_0}^{x} f(x_0,y_0)\,dx$$

$$f(x,y_0) = x-y_0$$

$$= x-1,\ x_0=0$$

$$y^{(1)} = 1+\int_0^x (x-1)\,dx$$

$$= 1+\left[\frac{x^2}{2}-x\right]_0^x$$

$$= 1+\frac{x^2}{2}-x$$

$$y^{(1)} = 1-x+\frac{x^2}{2} \rightarrow (2)$$

$$y^{(2)} = y_0+\int_{x_0}^{x} f(x,y^{(1)})\,dx$$

$$f(x,y^{(1)}) = x-y^{(1)}$$

$$= x-\left(1-x+\frac{x^2}{2}\right)$$

$$= x-1+x-\frac{x^2}{2}$$

$$= 2x-1-\frac{x^2}{2}$$

$$y^{(2)} = 1+\int_0^x \left(-1+2x-\frac{x^2}{2}\right)dx$$

$$= 1+\left[-x+\frac{2x^2}{2}-\frac{1}{2}\left(\frac{x^3}{3}\right)\right]_0^x$$

$$y^{(2)} = 1-x+x^2-\frac{x^3}{6} \rightarrow (3)$$

$$y^{(3)} = y_0+\int_{x_0}^{x} f(x,y^{(2)})\,dx$$

$$f(x,y^{(2)}) = x-y^{(2)}$$

$$= x-\left(1-x+x^2-\frac{x^3}{36}\right)$$

$$= x-1+x-x^2+\frac{x^3}{6}$$

$= -1+2x-x^2+\frac{x^3}{6}$

$y^{(3)} = -1+2x-x^2+\frac{x^3}{6} \rightarrow$ (4)

$y^{(3)} = 1+\int_0^x \left(-1+2x-x^2+\frac{x^3}{6}\right) dx$

$= 1+\left[-x+\frac{2x^2}{2}-\frac{x^3}{3}+\frac{1}{6}\frac{x^4}{4}\right]_0^x$

$y^{(3)} = 1-x+x^2-\frac{x^3}{3}+\frac{x^4}{24}$

$y^{(4)} = y_0+\int_{x_0}^x f(x, y^{(3)})\, dx$

$f(x, y^{(3)}) = x-\left(1-x+x^2-\frac{x^3}{3}+\frac{x^4}{24}\right)_0^x$

$= x-1+x-x^2+\frac{x^3}{3}-\frac{x^4}{24}$

$y = 2x-x^2-1+\frac{x^3}{3}-\frac{x^4}{24}$

$y^{(4)} = 1+\int_0^x \left(2x-x^2-1+\frac{x^3}{3}-\frac{x^4}{24}\right) dx$

$= 1+\left[\frac{2x^2}{2}-\frac{x^3}{3}+x+\frac{x^4}{4\cdot 3}-\frac{x^5}{5\times 24}\right]_0^x$

$y^{(4)} = 1+x^2-\frac{x^3}{3}+x+\frac{x^4}{12}-\frac{x^5}{120}$

$\therefore y = 1-x+x^2-\frac{x^3}{3}\cancel{-x}+\frac{x^4}{12}-\frac{x^5}{120}$

at $x = 0.1$

$y = 1-0.1+(0.1)^2-\frac{(0.1)^3}{3}+\frac{(0.1)^4}{12}-\frac{(0.1)^5}{120}$

$= +0.9+0.01-\frac{0.001}{3}+\frac{0.0001}{12}-\frac{0.00001}{120}$

$= 0.91 \cancel{-0.9096} + -0.000333+0.00000083-0.0000000833$

$= 0.90967522$

$y(0.1) = 0.9097$

at $x = 0.2$

$y = 1-0.2+(0.2)^2-\frac{(0.2)^3}{3}+\frac{(0.2)^4}{12}-\frac{(0.2)^5}{120}$

$= 0.8+0.04-\frac{0.008}{3}+\frac{0.00016}{12}-\frac{0.00032}{120}$

$= 0.84-0.0026667+0.0001333-0.0000026667$

$y(0.2) = 0.84-0.0026667+0.0001333-0.000002667$

$= 0.8375$

2. If $\frac{dy}{dx} = \frac{y-x}{y+x}$, find the value of y and $x=0.1$ using picards method. given that $y(0)=1$

solu/ Given that

$$\frac{dy}{dx} = \frac{y-x}{y+x}$$

$$f(x,y) = \frac{y-x}{y+x} \rightarrow (1)$$

Given $y(0)=1 \Rightarrow x_0=0,\ y_0=1$

By picards method

$$y^{(1)} = y_0 + \int_{x_0}^{x} f(x, y_0)\, dx$$

$$f(x, y_0) = \frac{y_0 - x}{y_0 + x} = \frac{1-x}{1+x}$$

$1-x=t$
$-1=dt$
$-dx=dt$

$$y^{(1)} = 1 + \int_0^x \left[\frac{1-x}{1+x}\right] dx$$

$$= 1 + \left[\frac{(1+x)(-1)-(1-x)(1)}{(1+x)^2}\right]_0^x = \cancel{1+}$$

put $1+x=t$

$1\,dx = dt$

$$= 1 + \left[\frac{-1-\cancel{x}-1+\cancel{x}}{(1+x)^2}\right]_0^x$$

lower limit $x=0$, $t=1+x$

$t=1$

$$= 1 + \left[-2\right]$$

$x=x$, $t=1+x$

$$= 1 + \int_1^{1+x} \frac{1-(t-1)}{t}\, dt$$

$$= 1 + \int_1^{1+x} \frac{\cancel{1}-t+\cancel{1}}{t}\, dt$$

$$= 1 + \int_1^{1+x} \frac{2-t}{t}\, dt$$

$$= 1 + \int_1^{1+x} \left[\frac{2}{t} - 1\right] dt$$

$$= 1 + \left[2\log t - t\right]_1^{1+x}$$

$$= 1 + \left[2\log(1+x) - (1+x)\right] - \left[2\log(1) - 1\right]$$

$$= 1 + \left[2\log(1+x) - (1+x)\right] + 1$$

$= 1+2\log(1+x) \cancel{-x} - x \cancel{+x}$

$y^{(1)} = 1-x+2\log(1+x)$

$y^{(2)} = y_0 + \int_{x_0}^{x} f(x, y^{(1)})\, dx$

$$f(x, y^{(1)}) = \frac{y^{(1)} - x}{y^{(1)} + x}$$

$$= \frac{1-x+2\log(1+x) - x}{1-\cancel{x}+2\log(1+x) \cancel{+x}}$$

$$= \frac{1-2x+2\log(1+x)}{1+2\log(1+x)}$$

$$y^{(2)} = 1 + \int_0^x \frac{1-2x+2\log(1+x)}{1+2\log(1+x)}\, dx$$

It is not defined

The solution of the given differential Equation is

$y = 1-x+2\log(1+x)$

put $x = 0.1$

$y = 1-(0.1)+2\log(1+0.1)$

$y = 0.9+2\log(1.1)$

$y = 0.9+2(0.0953)$

$y = 0.9+0.19062 = 0.\cancel{99}06$

$y = 1.0906$

Date 17/7/18

3\. Find the solution of $\frac{dy}{dx} = 1+xy$, $y(0)=1$ in the intraval $(0, 0.05)$ correct to three decimal places taking $h=0.1$

Solu Given $\frac{dy}{dx} = 1+xy \Rightarrow f(x,y) = 1+xy \rightarrow ①$

$y(0)=1, \quad x_0=0, \quad y_0=1, \quad h=0.1$

By picards method

$y^{(1)} = y_0 + \int_{x_0}^{x} f(x, y_0^{0})\, dx$

$f(x, y^{0}) = 1+x\cdot y_0', \quad y_0 = 1$

$= 1+x$

$y^{(1)} = 1 + \int_0^x (1+x)\, dx$

$$= 1 + \left[x + \frac{x^2}{2}\right]_0^1$$

$$= 1 + \left[x + \frac{x^2}{2}\right]$$

$$y^{(1)} = 1 + x + \frac{x^2}{2} \rightarrow (2)$$

$$y^{(2)} = y_0 + \int_{x_0}^{x} f(x, y^{(1)})\, dx$$

$$f(x, y^{(1)}) = 1 + xy^{(1)}$$

$$= 1 + x\left[1 + x + \frac{x^2}{2}\right]$$

$$= 1 + x + x^2 + \frac{x^3}{2}$$

$$y^{(2)} = 1 + \int_0^x \left[1 + x + x^2 + \frac{x^3}{2}\right] dx$$

$$= 1 + \left[x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{8}\right]_0^x$$

$$y^{(2)} = 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{8} \rightarrow (3)$$

$$y^{(3)} = y_0 + \int_{x_0}^{x} f(x, y^{(2)})\, dx$$

$$f(x, y^{(2)}) = 1 + xy^{(2)} = 1 + x\left[1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{8}\right]$$

$$= 1 + x + x^2 + \frac{x^3}{2} + \frac{x^4}{3} + \frac{x^5}{8}$$

$$y^{(3)} = 1 + \int_0^x \left[1 + x + x^2 + \frac{x^3}{2} + \frac{x^4}{3} + \frac{x^5}{8}\right] dx$$

$$= 1 + \left[x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{2 \times 4} + \frac{x^5}{3 \times 5} + \frac{x^6}{6 \times 8}\right]$$

$$y^{(3)} = 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{8} + \frac{x^5}{15} + \frac{x^6}{48}$$

$$y^{(4)} = y_0 + \int_{x_0}^{x} f(x, y^{(3)})\, dx$$

Not defined

$$\therefore y = 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{8} + \frac{x^5}{15} + \frac{x^6}{48}$$

$x_1 = x_0 + h$
$= 0 + 0.1$
$= 0.1$

$$y(0.1) = 1 + 0.1 + \frac{(0.1)^2}{2} + \frac{(0.1)^3}{3} + \frac{(0.1)^4}{8} + \frac{(0.1)^5}{15} + \frac{(0.1)^6}{48}$$

$$y(0.1) = 1 + 0.1 + \frac{0.01}{2} + \frac{0.001}{3} + \frac{0.0001}{8} + \frac{0.00001}{15} + \frac{0.000001}{48}$$

$$= 1.1 + 0.005 + 0.00033 + 0.000025 + 0.00000067$$

$$+ 0.000000021$$

$$y(0.1) = 1.105343191$$

$$y(0.1) = 1.105$$

$x_2 = x_1 + h$
$= 0.1 + 0.1$
$= 0.2$

$$y(0.2) = 1 + 0.2 + \frac{(0.2)^2}{2} + \frac{(0.2)^3}{3} + \frac{(0.2)^4}{8} + \frac{(0.2)^5}{15} + \frac{(0.2)^6}{48}$$

$$y(0.2) = 1.2 + \frac{0.04}{2} + \frac{0.008}{3} + \frac{0.0016}{8} + \frac{0.00032}{15} + \frac{0.0000064}{48}$$

$$= 1.2 + 0.02 + 0.00267 + 0.0002 + 0.0000213 + 0.00000133$$

$$= 1.22289263$$

$$= 1.223$$

$x_3 = x_2 + h$
$= 0.2 + 0.1$
$= 0.3$

$$y(0.3) = 1 + 0.3 + \frac{(0.3)^2}{2} + \frac{(0.3)^3}{3} + \frac{(0.3)^4}{8} + \frac{(0.3)^5}{15} + \frac{(0.3)^6}{48}$$

$$y(0.3) = 1.3 + \frac{0.03}{2} + \frac{0.027}{3} + \frac{0.0081}{8} + \frac{0.00243}{15} + \frac{0.000729}{48}$$

$$= 1.3 + 0.045 + 0.009 + 0.00101 + 0.000162 + 0.0000151$$

$$= 1.3551871$$

$$y(0.3) = 1.355$$

$x_4 = x_3 + h$
$= 0.3 + 0.1$
$= 0.4$

$$y(0.4) = 1 + 0.4 + \frac{(0.4)^2}{2} + \frac{(0.4)^3}{3} + \frac{(0.4)^4}{8} + \frac{(0.4)^5}{15} + \frac{(0.3)^6}{48}$$

$$y(0.4) = 1 + 0.4 + \frac{0.96}{2} + \frac{0.064}{3} + \frac{0.0256}{8} + \frac{0.01024}{15} + \frac{0.004096}{48}$$

$$= 1.4 + 0.08 + 0.02133 + 0.0032 + 0.0006826 + 0.00008$$

$$= 1.505265$$

$$y(0.4) = 1.505$$

$x_5 = x_4 + h$ $\quad y(0.5) = 1 + 0.5 + \frac{(0.5)^2}{2} + \frac{(0.5)^3}{3} + \frac{(0.5)^4}{8} + \frac{(0.5)^5}{15} + \frac{(0.5)^6}{48}$

$= 0.4 + 1$

$= 0.5$

$$y(0.5) = 1.5 + \frac{0.25}{2} + \frac{0.125}{3} + \frac{0.0625}{8} + \frac{0.03125}{15} + \frac{0.015625}{48}$$

$$= 1.5 + 0.125 + 0.04167 + 0.0078125 + 0.0020833 + 0.00032$$

$$= 1.6798155$$

$$y(0.5) = 1.68$$

4. For the differential Equation $\frac{dy}{dx} = x - y^2$, $y(0) = 0$ Calculate $y(0.2)$ by using picards method to third approximation and round of the value into four decimal places

Solu) Given $\frac{dy}{dx} = x - y^2$

$$f(x,y) = x - y^2 \rightarrow ①$$

$$y(0) = 0, \quad x_0 = 0, \quad y_0 = 0$$

By picards method

$$y^{(1)} = y_0 + \int_{x_0}^{x} f(x, y_0)\, dx$$

$$f(x, y_0) = x - y_0^2$$

$$= x - 0$$

$$= x$$

$$y^{(1)} = 0 + \int_0^x x\, dx$$

$$y^{(1)} = \left[\frac{x^2}{2}\right]_0^x = \frac{x^2}{2} \rightarrow ②$$

$$y^{(2)} = y_0 + \int_{x_0}^{x} f(x, y^{(1)})\, dx$$

$$f(x, y^{(1)}) = x - y_1^2$$

$$= x - \left(\frac{x^2}{2}\right)^2$$

$$= x - \frac{x^4}{4}$$

$$y^{(2)} = 0 + \int_0^x \left[x - \frac{x^4}{4}\right] dx$$

$$y^{(2)} = \frac{x^2}{2} - \frac{x^5}{20} \rightarrow ③$$

$$y^{(3)} = y_0 + \int_{x_0}^{x} f(x, y^{(2)})\,dx$$

$$f(x, y^{(2)}) = x - (y_{(2)})^2$$

$$= x - \left[\frac{x^2}{2} - \frac{x^5}{20}\right]^2$$

$$= x - \left[\frac{x^4}{4} + \frac{x^{10}}{400} - \frac{\cancel{2}x^7}{\cancel{40}\,20}\right]$$

$$= x - \frac{x^4}{4} - \frac{x^{10}}{400} + \frac{x^7}{20}$$

$$y^{(3)} = 0 + \int_0^x \left[x - \frac{x^4}{4} - \frac{x^{10}}{400} + \frac{x^7}{20}\right]$$

$$y^{(3)} = \left[\frac{x^2}{2} - \frac{x^5}{20} - \frac{x^{11}}{4400} + \frac{x^8}{160}\right] \rightarrow (4)$$

$$y = \frac{x^2}{2} - \frac{x^5}{20} - \frac{x^{11}}{4400} + \frac{x^8}{160}$$

$$y(0.2) = \frac{(0.2)^2}{2} - \frac{(0.2)^5}{20} - \frac{(0.2)^{11}}{4400} + \frac{(0.2)^8}{160}$$

$$= \frac{0.04}{2} - \frac{0.00032}{20} - \frac{0.0000000204\!\!\!\!\!\!\!\!\!\!\;\;\;\;\;\;\;\;\;\;\;\;}{4400} + \frac{0.00000256}{160}$$

$$= 0.02 - 0.000016 - 0.000000000004654$$

$$+ 0.000000016$$

$$= 0.019984016$$

$$y(0.2) = 0.02$$

5. find an approximate value of y when $x = 0.1$, if $\frac{dy}{dx} = x - y^2$ and $y = 1$, at $x = 0$ using picards method upto ~~three~~ two approximations

Sol: Given that

$$\frac{dy}{dx} = x - y^2$$

$$f(x, y) = x - y^2 \rightarrow (1)$$

$$y = 1, \quad x = 0.1, \quad x_0 = 0, \quad y_0 = 1$$

By picards method

$$y^{(1)} = y_0 + \int_{x_0}^{x} f(x, y_0)\,dx$$

$$f(x, y_0) = x - y_0^2$$

$$= x - 1$$

$= x-1$

$y^{(1)} = 1 + \int_0^x (x-1)\,dx$

$y(1) = 1 + \left[\frac{x^2}{2} - x\right] \rightarrow 2$

$y(2) = y_0 + \int_{x_0}^{x} f(x, y^{(1)})\,dx$

$f(x, y^{(1)}) = x - y_1^2$

$= x - \left[1 + \frac{x^2}{2} - x\right]^2$

$= x - 1 - \frac{x^2}{2} + x$

$= 2x - 1 - \frac{x^2}{2}$

$y(2) = 1 + \int_0^x \left[2x - 1 - \frac{x^2}{2}\right]$

$= 1 + 2\frac{x^2}{2} - x - \frac{x^3}{6}$

$y(2) = 1 + x^2 - x - \frac{x^3}{6}$

$y = 1 - x + x^2 - \frac{x^3}{6}$

$y(0.1) = 1 - 0.1 + (0.1)^2 - \frac{(0.1)^3}{6}$

$= 1 - 0.1 + 0.01 - \frac{0.001}{6}$

$= 1 - 0.1 + 0.01 - 1.00016$

$= -0.09016$

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$= x - \left[1 + \frac{x^4}{4} - x^2 + 2\frac{x^2}{2} - 2\frac{x^2}{2}x - 2x\right]$

$= x - \left[1 + \frac{x^4}{4} + x^2 + x^2 - x^3 - 2x\right]$

$= x - \left[1 + 2x^2 + \frac{x^4}{4} - x^3 - 2x\right]$

$= x - \left[1 + 2x^2 - 2x + \frac{x^4}{4} - x^3\right]$

$y(2) = 1 + \int_0^x \left[x - 1 - 2x^2 + 2x - \frac{x^4}{4} + x^3\right]$

$y(2) = 1 + \frac{x^2}{2} - x - 2\frac{x^3}{3} + \frac{2x^2}{2} - \frac{x^5}{4\times5} + \frac{x^4}{4}$

$y(2) = 1 + \frac{x^2}{2} - x - \frac{2}{3}x^3 + x^2 - \frac{x^5}{20} + \frac{x^4}{4}$

$y(0.1) = 1 + \frac{(0.1)^2}{2} - 0.1 - \frac{2}{3}(0.1)^3 + (0.1)^2 - \frac{(0.1)^5}{20} + \frac{(0.1)^4}{4}$

$= 1 + \frac{0.01}{2} - 0.1 - \frac{2}{3}(0.001) + 0.01 - \frac{0.00001}{20} + 0.0001$

$= 0.9101 + 0.0005 - (0.6667)(0.001)$

0.99328

c)

$= 0.2 + 0.02 + 0.00013$

$= 0.22013$

$y(0.4) = 0.4 + \frac{(0.4)^2}{2} + \frac{(0.4)^4}{12}$

$= 0.4 + \frac{0.16}{2} + \frac{0.0256}{12}$

$= 0.4 + 0.08 + 0.00213 =$

$= 0.48213$

1/8/18

7. R-K Method of 4^{th} order

Consider $\frac{dy}{dx} = f(x, y)$

$$y_1 = y_0 + k$$

$$K = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$k_1 = h \cdot f(x_0, y_0)$$

$$k_2 = h \cdot f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$k_3 = h \cdot f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$k_4 = h \cdot f(x_0 + h, y_0 + k_3)$$

Similarly

$$y_2 = y_1 + K$$

$$K = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$k_1 = h \cdot f(x_1, y_1)$$

$$k_2 = h \cdot f\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right)$$

$$k_3 = h \cdot f\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right)$$

$$k_4 = h \cdot f(x_1 + h, y_1 + k_3)$$

1. Use R-K method of 4th order. Find the value of y at $x=0.1$ given $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$,

Solu) Given

$$\frac{dy}{dx} = \frac{y-x}{y+x}$$

$$\Rightarrow f(x,y) = \frac{y-x}{y+x} \longrightarrow (1)$$

$y(0)=1 \Rightarrow x_0=0; \; y_0=1 \; ; \; h=0.1$

$$k_1 = h.f(x_0, y_0)$$

$$= h.f(0,1)$$

$$= 0.1 \frac{1-0}{1+0}$$

$$= 0.1 \times 1 = 0.1$$

$$\boxed{k_1 = 0.1}$$

$$k_2 = h.f\left(x_0+\frac{h}{2}, y_0+\frac{k_1}{2}\right)$$

$$= h.f\left(0+\frac{0.1}{2}, 1+\frac{0.1}{2}\right)$$

$$= h f\left(\frac{0.1}{2}, \frac{2.1}{2}\right)$$

$$= 0.1 f(0.05, 1.05)$$

$$= 0.1 \left[\frac{1.05-0.05}{1.05+0.05}\right]$$

$$= 0.1 \times \frac{1}{1.1}$$

$$\boxed{k_2 = 0.0909}$$

$$k_3 = h.f\left(x_0+\frac{h}{2}, y_0+\frac{k_2}{2}\right)$$

$$= h.f\left(0+\frac{0.1}{2}, 1+\frac{0.0909}{2}\right)$$

$$= h.f(0.05, 1+0.045454)$$

$$= h.f(0.05, 1.045454)$$

$$= 0.1 \left[\frac{1.045454-0.05}{1.045454+0.05}\right]$$

$$= 0.1 \times \frac{0.995454}{1.095454}$$

$= 0.09087133$

$k_3 = 0.0909$

$K_4 = h.f(x_0+h, y_0+K_3)$

$= h.f(0+0.1, 1+0.0909)$

$= h.f(0.1, 1.0909)$

$= 0.1\left[\frac{1.0909-0.1}{1.0909+0.1}\right]$

$= 0.1 \times \frac{0.9909}{1.1909}$

$\boxed{K_4 = 0.0832}$

$K = \frac{1}{6}[K_1+2K_2+2K_3+K_4]$

$= \frac{1}{6}[0.1+2(0.0909)+2(0.0909)+0.0832]$

$= \frac{1}{6}[0.1+0.1818+0.1818+0.0832]$

$= \frac{0.5468}{6}$

$K = 0.09113$

$K = 0.0911$

2. $y_1 = y_0+K$

$= 1+0.0911$

$y_1 = 1.0911, \quad x_1 = 0.1$

2. Use R.K. method of 4^{th} order to find the value of y at $x=0.1$, given $y' = xy+1$, $y(0)=1$

<u>solu</u> Given $\frac{dy}{dx} = xy+1 = y'$

$f(x,y) = xy+1 \rightarrow (1)$

$y(0)=1, \quad x_0=0, \quad y_0=1, \quad h=0.1$

$K_1 = h.f(x_0,y_0)$

$= hf(0,1)$

$= h[0(1)+1]$

$= 0.1 \times 1$

$\boxed{k_1 = 0.1}$

$k_2 = h \cdot f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$= hf\left(0 + \frac{0.1}{2}, 1 + \frac{0.1}{2}\right)$

$= hf(0.05, 1+0.05)$

$= hf(0.05, 1.05)$

$= 0.1\left[(0.05)(1.05) + 1\right]$

$= 0.1\left[1.0525\right]$

$k_2 = 0.10525$

$\boxed{k_2 = 0.1053}$

$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$= hf\left(0 + \frac{0.1}{2}, 1 + \frac{0.10523}{2}\right)$

$= hf(0.05, 1+0.05265)$

$= hf(0.05, 1.05265)$

$= 0.1\left[(0.05)(1.05265) + 1\right]$

$= 0.1\left[1.0526325\right]$

$k_3 = 0.10526325 = 0.1053$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$= hf(0+0.1, 1+0.1053)$

$= hf(0.1, 1.1053)$

$= h\left[(0.1)(1.1053) + 1\right]$

$= 0.1\left[1.11053\right]$

$= 0.111053$

$k_4 = 0.1111$

$k = \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$

$= \frac{1}{6}\left[0.1 + 2(0.10525) + 2(0.1053) + 0.1111\right]$

$= \frac{1}{6}[0.1 + 0.2106 + 0.2106 + 0.1111]$

$= \frac{0.6323}{6}$

$= 0.10538$

$K = 0.1054$

$y_1 = y_0 + K$

$= 1 + 0.1054$

$y_1 = 1.1054 \; ; \; x_1 = 0.1$

Date 2/8/18

3. Using R.K method of 4^{th} order find y when $x = 0.1$ and 0.2, Given that $x = 0$, when $y = 1$ and $\frac{dy}{dx} = x + y$

Solu) Given

$\frac{dy}{dx} = x + y$

$f(x, y) = x + y \rightarrow (1)$

$x = 0, \; y = 1, \; x_0 = 0, \; y_0 = 1, \; h = 0.1$

Case(i)

$K_1 = h \cdot f(x_0, y_0)$

$x_1 = x_0 + h$

$= 0.1 + 0.1$

$= 0.2$

$K_1 = h \cdot f(0, 1)$

$= h \cdot [0 + 1]$

$K_1 = 0.1$

$K_2 = h \cdot f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$

$= h \cdot f\left(0 + \frac{0.1}{2}, 1 + \frac{0.1}{2}\right)$

$= h \cdot f(0.05, 1 + 0.05)$

$= h \cdot f(0.05, 1.05)$

$= 0.1[0.05 + 1.05]$

$= 0.1[1.1]$

$K_2 = 0.$

$k_3 = h \cdot f\left(x_0 + \frac{h}{2},\ y_0 + \frac{k_2}{2}\right)$

$= h \cdot f\left(0 + \frac{0.1}{2},\ 1 + \frac{0.11}{2}\right)$

$= h \cdot f(0.05,\ 1 + 0.055)$

$= h \cdot f(0.05,\ 1.055)$

$= 0.1\,[0.05 + 1.055]$

$= 0.1\,[1.105]$

$k_3 = 0.1105$

$k_4 = h \cdot f(x_0 + h,\ y_0 + k_3)$

$= h \cdot f(0 + 0.1,\ 1 + 0.1105)$

$= h \cdot f(0.1,\ 1.1105)$

$= 0.1\,[0.1 + 1.1105]$

$= 0.1\,[1.2105]$

$k_4 = 0.12105 = 0.1211$

$K = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= \frac{1}{6}[0.1 + 2(0.11) + 2(0.1105) + 0.12105]$

$= \frac{1}{6}[0.1 + 0.22 + 0.221 + 0.12198]$

$= \frac{1}{6}[0.6621]$

$= 0.0736833 \quad 0.11035$

$K = 0.1104$

$y_1 = y_0 + K$

$= 1 + 0.1104$

$= 1.1104 \quad x_1 = 0.1$

case (ii)

$x_2 = x_1 + h$

$= 0.1 + 0.1$

$= 0.2$

$k_1 = h.f(x_1, y_1)$

$= h.f(0.1, 1.1104)$

$= h[0.1 + 1.1104]$

$= h[1.2104]$

$= 0.1 \times 1.2104$

$k_1 = 0.12104 = 0.121$

$k_2 = h.f(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2})$

$= h.f(0.1 + \frac{0.1}{2}, 1.1104 + \frac{0.121}{2})$

$= h.f(0.1 + 0.05, 1.1104 + 0.0605)$

$= h.f(0.15, 1.1709)$

$= 0.1[0.15 + 1.1709]$

$= 0.1[1.3209]$

$= 0.13209$

$k_2 = 0.1321$

$k_3 = h.f(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2})$

$= h.f(0.1 + \frac{0.1}{2}, 1.1104 + \frac{0.1321}{2})$

$= h.f(0.1 + 0.05, 1.1104 + 0.06605)$

$= h.f(0.15, 1.17645)$

$= 0.1[0.15 + 1.17645]$

$= 0.1[1.32645]$

$k_3 = 0.132645 = 0.1327$

$k_4 = h.f(x_1 + h, y_1 + k_3)$

$= h.f(0.1 + 0.1, 1.1104 + 0.1327)$

$= h.f(0.2, 1.2431)$

$= 0.1[0.2 + 1.2431]$

$= 0.1[1.4431]$

$= 0.14431$

$K_4 = 0.1443$

$K = \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$

$= \frac{1}{6}[0.121 + 2(0.1321) + 2(0.1327) + 0.1443]$

$= \frac{1}{6}[0.121 + 0.2642 + 0.2654 + 0.1443]$

$= \frac{1}{6}[0.7949]$

$= 0.132483$

$K = 0.13249$

$y_2 = y_1 + K$

$= 1.1104 + 0.13249$

$= 1.24289$

$y_2 = 1.2429$

4. Use R-K method of 4th order to find y when $x = 1.2$ given $\frac{dy}{dx} = x^2 + y^2$, $y(1) = 1.5$

solu) Given that

$\frac{dy}{dx} = x^2 + y^2$

$f(x_0, y_0) = x^2 + y^2 \rightarrow (1)$

$y(1) = 1.5, \quad x_0 = 1, \quad y_0 = 1.5, \quad h = 0.1$

Case (i) $x_1 = x_0 + h$

$= 1 + 0.1$

$= 1.1$

$K_1 = h \cdot f(x_0, y_0)$

$= h \cdot f(1, 1.5)$

$= h \cdot [1 + (1.5)^2]$

$= 0.1[1 + 2.25]$

$= 0.1[3.25]$

$K_1 = 0.325$

$k_2 = h \cdot f\left(x_0 + \frac{h}{2},\ y_0 + \frac{k_1}{2}\right)$

$= h \cdot f\left(1 + \frac{0.1}{2},\ 1.5 + \frac{0.325}{2}\right)$

$= h \cdot f(1 + 0.05,\ 1.5 + 0.1625)$

$= h \cdot f(1.05,\ 1.6625)$

$= 0.1\left[(1.05)^2 + (1.6625)^2\right]$

$= 0.1[1.1025 + 2.76390]$

$= 0.1[3.8664]$

$= 0.38664$

$k_2 = 0.3866$

$k_3 = h \cdot f\left(x_0 + \frac{h}{2},\ y_0 + \frac{k_2}{2}\right)$

$= h \cdot f\left(1 + \frac{0.1}{2},\ 1.5 + \frac{0.3866}{2}\right)$

$= h \cdot f(1 + 0.05,\ 1.5 + 0.1933)$

$= h \cdot f(1.05,\ 1.6933)$

$= 0.1\left[(1.05)^2 + (1.6933)^2\right]$

$= 0.1[1.1025 + 2.8673]$

$= 0.1[3.9698]$

$= 0.39698$

$k_3 = 0.397$

$k_4 = h \cdot f(x_0 + h,\ y_0 + k_3)$

$= h \cdot f(1 + 0.1,\ 1.5 + 0.397)$

$= h \cdot f(1.1,\ 1.897)$

$= 0.1\left[(1.1)^2 + (1.897)^2\right]$

$= 0.1[1.21 + 3.599]$

$= 0.1[4.809]$

$k_4 = 0.4809$

$K = \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$

$= \frac{1}{6}[0.325 + 2(0.3866) + 2(0.397) + 0.4809]$

$= \frac{1}{6}[0.325 + 0.7732 + 0.794 + 0.4809]$

$= \frac{1}{6}[2.3731]$

$= 0.395516$

$K = 0.3956$

$y_1 = y_0 + K$

$= 1.5 + 0.3956$

$y_1 = 1.8956$, $x_1 = 1.1$ pending

case (ii)

$x_2 = x_1 + h$

$= 1.1 + 0.1$

$= 1.2$

$K_1 = h.f(x_q, y_q)$

$= h.f(\cancel{1.1, 1.5})$

$= h.f(1.1, 1.8956)$

$= h.f[(1.1)^2 + (1.8956)^2]$

$= 0.1[1.21 + 3.5933]$

$= 0.1[4.8033]$

$K_1 = 0.48033$

$K_2 = h.f\left(x_q + \frac{h}{2}, y_1 + \frac{K_1}{2}\right)$

$= h.f\left(1.1 + \frac{0.1}{2}, 1.8956 + \frac{0.48033}{2}\right)$

$= h.f(1.1 + 0.05, 1.8956 + 0.240165)$

$= h.f(1.15, 2.13577)$

$= 0.1[(1.15)^2 + (2.13577)^2]$

$= 0.1[1.3225 + 4.5615]$

$= 0.1[5.884]$

$k_2 = 0.5884$

$k_3 = h.f\left(x_1 + \frac{h}{2}, \; y_1 + \frac{k_2}{2}\right)$

$= h.f\left(1.1 + \frac{0.1}{2}, \; 1.8956 + \frac{0.5884}{2}\right)$

$= h.f(1.1 + 0.05, \; 1.8956 + 0.2942)$

$= h.f(1.15, \; 2.1898)$

$= 0.1[(1.15)^2 + (2.1898)^2]$

$= 0.1[1.3225 + 4.7953]$

$= 0.1[6.1178]$

$k_3 = 0.61178 = 0.6118$

$k_4 = h.f(x_1 + h, \; y_1 + k_3)$

$= h.f(1.1 + 0.1, \; 1.8956 + 0.6118)$

$= h.f(1.2, \; 2.5074)$

$= 0.1[(1.2)^2 + (2.5074)]$

$= 0.1[1.44 + 6.28705]$

$= 0.1[7.72705]$

$k_4 = 0.7727$

$k = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= \frac{1}{6}[0.48033 + 2(0.5884) + 2(0.6118) + 0.7727]$

$= \frac{1}{6}[0.48033 + 1.1768 + 1.2236 + 0.7727]$

$= \frac{1}{6}[3.65343]$

$= 0.608905$

$k = 0.6089$

$y_2 = y_1 + K$

$= 1.8956 + 0.6089$

$y_2 = 2.5045, \ x_2 = 1.2.$

Date 3/8/18

4. Given the initial value problem $y' = 1+y^2$, $y(0)=0$, find $y(0.6)$ by R.K method of 4th order taking $h=0.2$

solu) Given

$y' = 1+y^2$

$f(x_0, y_0) = 1+y^2 \rightarrow (1)$

$y(0)=0, \ x_0=0, \ y_0=0$

case (i)

$x_1 = x_0 + h$

$= 0 + 0.2$

$= 0.2$

$K_1 = hf(x_0, y_0)$

$= h.f(0, 0)$

$= 0.2[1+0^2]$

$K_1 = 0.2$

$K_2 = h.f(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2})$

$= h.f(0 + \frac{0.2}{2}, 0 + \frac{0.2}{2})$

$= h.f(0.1, 0.1)$

$= 0.2[1+(0.1)^2]$

$= 0.2[1+0.01]$

$= 0.2[1.01]$

$K_2 = 0.202$

$K_3 = h.f(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2})$

$= h.f(0 + \frac{0.2}{2}, 0 + \frac{0.202}{2})$

$= h.f(0.1, 0.101)$

$= 0.2[1+(0.101)^2]$

$= 0.2[1+0.010201]$

$= 0.2[1.010201]$

$$\begin{array}{r} 0.1 \\ 0.1 \\ \hline 01 \\ 00 \end{array}$$

$k_3 = 0.2020402$

$k_3 = 0.202$

$k_4 = h \cdot f(x_0+h,\ y_0+k_3)$

$= h \cdot f(0+0.2,\ 0+0.2020)$

$= h \cdot f(0.2,\ 0.2020)$

$= 0.2[1+(0.202)^2]$

$= 0.2[1+0.0408004]$

$= 0.2[1.0408004]$

$= 0.2081608$

$k_4 = 0.2082$

$K = \frac{1}{6}[k_1+2k_2+2k_3+k_4]$

$= \frac{1}{6}[0.2+2(0.202)+2(0.202)+0.2082]$

$= \frac{1}{6}[0.2+0.404+0.404+0.2082]$

$= \frac{1}{6}[1.2162]$

$K = 0.2027$

$y_1 = y_0 + K$

$= 0+0.2027$

$y_1 = 0.2027 \quad x_1 = 0.2$

case (ii)

$x_2 = x_1 + h$

$= 0.2+0.2$

$x_2 = 0.4$

$k_1 = h \cdot f(x_1, y_1)$

$= h \cdot f(0.2,\ 0.2027)$

$= h \cdot [1+(0.2027)^2]$

$= 0.2[1+0.04108729]$

$= 0.2[1.04108729]$

$k_1 = 0.2082$

$$K_2 = h \cdot f\left(x_1 + \frac{h}{2},\ y_1 + \frac{K_1}{2}\right)$$

$$= h \cdot f\left(0.2 + \frac{0.2}{2},\ 0.2027 + \frac{0.2082}{2}\right)$$

$$= h \cdot f(0.2 + 0.1,\ 0.2027 + 0.1041)$$

$$= h \cdot f(0.3,\ 0.3068)$$

$$= 0.2[1 + (0.3068)^2]$$

$$= 0.2[1 + 0.09412624]$$

$$= 0.2[1.09412624]$$

$$= 0.218825248$$

$$K_2 = 0.2188$$

$$K_3 = h \cdot f\left(x_1 + \frac{h}{2},\ y_1 + \frac{K_2}{2}\right)$$

$$= h \cdot f\left(0.2 + \frac{0.2}{2},\ 0.2027 + \frac{0.2188}{2}\right)$$

$$= h \cdot f(0.2 + 0.1,\ 0.2027 + 0.10948)$$

$$= h \cdot f(0.3,\ 0.31218)$$

$$= 0.2[1 + (0.31218)^2]$$

$$= 0.2[1 + 0.097422015]$$

$$= 0.2[1.097422015]$$

$$= 0.219484403$$

$$K_3 = 0.2194$$

$$K_4 = h \cdot f(x_1 + h,\ y_1 + K_3)$$

$$= h \cdot f(0.2 + 0.2,\ 0.2027 + 0.2195)$$

$$= h \cdot f(0.4,\ 0.4222)$$

$$= 0.2[1 + (0.4222)^2]$$

$$= 0.2[1 + 0.17825284]$$

$$= 0.2[1.17825284]$$

$$= 0.235650568$$

$$K_4 = 0.2357$$

$K = \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$

$= \frac{1}{6}[0.2082 + 2(0.2188) + 2(0.2195) + 0.2357]$

$= \frac{1}{6}[0.2082 + 0.4376 + 0.439 + 0.2357]$

$= \frac{1}{6}[1.3205]$

$= 0.2200083333$

$K = 0.2209$

$y_2 = y_1 + K$

$= 0.2027 + 0.2201$

$y_2 = 0.4228 \quad x_2, 0.4$

case(ii)

$x_3 = x_2 + h$

$= 0.4 + 0.2$

$x_3 = 0.6$

$K_1 = h \cdot f(x_2, y_2)$

$= h \cdot f(0.4, 0.4228)$

$= 0.2[1 + (0.4228)^2]$

$= 0.2[1 + 0.17875984]$

$= 0.2[1.17875984]$

$= 0.235751968$

$K_1 = 0.2358$

$K_2 = h \cdot f(x_2 + \frac{h}{2}, y_2 + \frac{K_1}{2})$

$= h \cdot f(0.4 + \frac{0.2}{2}, 0.4228 + \frac{0.2358}{2})$

$= h \cdot f(0.5, 0.4228 + 0.1179)$

$= h \cdot f(0.5, 0.5407)$

$= 0.2[1 + (0.5407)^2]$

$= 0.2[1 + 0.29235649]$

$= 0.2[1.29235649]$

$= 0.258471298$

$K_2 = 0.2585$

$K_3 = h.f\left(x_2 + \frac{h}{2}, y_2 + \frac{K_2}{2}\right)$

$= h.f\left(0.4 + \frac{0.2}{2}, 0.4228 + \frac{0.2585}{2}\right)$

$= h.f(0.5, 0.4228 + 0.12925)$

$= h.f(0.5, 0.55205)$

$= 0.2\left[1 + (0.55205)^2\right]$

$= 0.2\left[1 + 0.3047592202\right]$

$= 0.2\left[1.304759203\right]$

$= 0.26095184$

$K_3 = 0.2601$

$K_4 = h.f(x_2 + h, y_2 + K_3)$

$= h.f(0.4 + 0.2, 0.4228 + 0.261)$

$= h.f(0.6, 0.6838)$

$= 0.2\left[1 + (0.6838)^2\right]$

$= 0.2\left[1 + 0.46758244\right]$

$= 0.2\left[1.46758244\right]$

$= 0.293516488$

$K_4 = 0.2935$

$K = \frac{1}{6}\left[K_1 + 2K_2 + 2K_3 + K_4\right]$

$= \frac{1}{6}\left[0.2358 + 2(0.2585) + 2(0.261) + 0.2935\right)$

$= \frac{1}{6}\left[0.2358 + 0.517 + 0.522 + 0.2935\right)$

$= \frac{1}{6}\left[1.5683\right]$

$= 0.26138333$

$K = 0.2614$

$y_3 = y_2 + K$

$= 0.4228 + 0.2614 = 0.6842$

5. find the value of $y(1.1)$ using R-K method of 4th order

HW Given that $\frac{dy}{dx} = 3x+y^2$, $y(1) = 1$

6 find the value of $y(1.1)$ using R-K method of 4th order

given $\frac{dy}{dx} = y^2+xy$, $y(1) = 1$

Solu Given that

$$\frac{dy}{dx} = 3x+y^2$$

$$f(x_0, y_0) = 3x+y^2 \rightarrow ①$$

$$y(1) = 1, \quad x_0 = 1, \quad y_0 = 1$$

$$k_1 = h.f(x_0, y_0)$$

$$= h.f(1, 1)$$

$$= 0.1[3(1)+1]$$

$$= 0.1[3+1]$$

$$k_1 = 0.1[4] = 0.4$$

$$k_2 = h.f(x_0+\frac{h}{2}, y_0+\frac{k_1}{2})$$

$$k_2 = h.f(1+\frac{0.1}{2}, 1+\frac{0.4}{2})$$

$$= h.f(1+0.05, 1+0.2)$$

$$= h.f(1.05, 1.2)$$

$$= 0.1[3(1.05)+(1.2)^2]$$

$$= 0.1[3.15+1.44]$$

$$= 0.1[4.59]$$

$$k_2 = 0.459$$

$$k_3 = h.f(x_0+\frac{h}{2}, y_0+\frac{k_2}{2})$$

$$= h.f(1+\frac{0.1}{2}, 1+\frac{0.459}{2})$$

$$= h.f(1.05, 1+0.2295)$$

$$= h.f(1.05, 1.2295)$$

$$= 0.1[3(1.05)+(1.2295)^2]$$

$$= 0.1[3.15+1.51167025]$$

$= 0.1[4.66167025]$

$= 0.466167025$

$= 0.4662$

$K_4 = h \cdot f(x_0 + h,\ y_0 + K_3)$

$= h \cdot f(1 + 0.1,\ 1 + 0.4662)$

$= h \cdot f(1.1,\ 1.4662)$

$= 0.1[3(1.1) + (1.4662)^2]$

$= 0.1[3.3 + 2.14974244]$

$= 0.1[5.44974244]$

$= 0.544974244$

$K_4 = 0.5441$

$K = \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$

$= \frac{1}{6}[0.4 + 2(0.459) + 2(0.4662) + 0.5441]$

$= \frac{1}{6}[0.4 + 0.918 + 0.9324 + 0.5441]$

$= \frac{1}{6}[2.7945]$

$= 0.46575$

$K = 0.4658$

$y_1 = y_0 + K$

$= 1 + 0.4658$

$= 1.4658$

6. Given that

$\frac{dy}{dx} = y^2 + xy$

$f(x_0, y_0) = y^2 + xy \rightarrow ①$

$y(1) = 1, \quad x_0 = 1, \quad y_0 = 1$

$K_1 = h \cdot f(x_0, y_0)$

$= h \cdot f(1, 1)$

$= 0.1[1 + 1(1)]$

$= 0.1 \times 2$

$K_1 = 0.2$

$$K_2 = h.f\left(x_0+\frac{h}{2},\ y_0+\frac{K_1}{2}\right)$$

$$= h.f\left(1+\frac{0.1}{2},\ 1+\frac{0.2}{2}\right)$$

$$= h.f(1+0.05,\ 1+0.1)$$

$$= h.f(1.05,\ 1.1)$$

$$= 0.1\left[(1.1)^2+(1.05)(1.1)\right]$$

$$= 0.1\left[1.21+1.155\right]$$

$$= 0.1\left[2.365\right]$$

$$K_2 = 0.2365$$

$$K_3 = h.f\left(x_0+\frac{h}{2},\ y_0+\frac{K_2}{2}\right)$$

$$= h.f\left(1+\frac{0.1}{2},\ 1+\frac{0.2365}{2}\right)$$

$$= h.f(1+0.05,\ 1+0.11825)$$

$$= h.f(1.05,\ 1.11825)$$

$$= 0.1\left[(1.11825)^2+(1.05)(1.11825)\right]$$

$$= 0.1\left[1.250483063+1.1741625\right]$$

$$= 0.1\left[2.424645563\right]$$

$$= 0.2424645563$$

$$K_3 = 0.2425$$

$$K_4 = h.f(x_0+h,\ y_0+K_3)$$

$$= h.f(1+0.1,\ 1+0.2425)$$

$$= h.f(1.1,\ 1.2425)$$

$$= 0.1\left[(1.2425)^2+(1.1)(1.2425)\right]$$

$$= 0.1\left[1.54380625+1.36675\right]$$

$$= 0.1\left[2.91055625\right]$$

$$= 0.291055625$$

$$K_4 = 0.2919$$

$$K = \frac{1}{6}\left[K_1+2K_2+2K_3+K_4\right]$$

$= \frac{1}{6}[0.2+2(0.2365)+2(0.2428)+0.2919]$

$= \frac{1}{6}[0.2+0.473+0.485+0.2911]$

$= \frac{1}{6}[1.4491]$

$= 0.2415166667$

$K = 0.2415$

$y_1 = y_0 + K$

$= 1 + 0.2415$

$y_1 = 1.2415$